ADJACENCY EIGENVALUES FOR UNDERLYING SPLIT MULTIGRAHPS

ANALISA ESPINO & JOHN T. SACCOMAN—DEPARTMENT OF MATHEMATICS, SETON HALL UNIVERSITY

ABSTRACT

A split graph is a graph whose vertices can be partitioned into a clique and an independent set (whose nodes are called cones). A split graph G is proper if every cone has the same degree. Most results in spectral graph theory do not address multigraph concerns. An exception is *Congressus Numerantium 230,* in which the Laplacian spectrum for multigraphs having underlying split graphs of a particular structure was presented. In this work, we present a conjecture for the adjacency spectrum of these graphs. Further, if these multigraphs model a satellite communications network, we conjecture a formula for the number of triangles between ground stations and orbiting satellites.

INTRODUCTION

Let G = (V,E) denote a graph with a set on n vertices and e edges. A graph G is called a *split graph* if its vertices can be partitioned into a clique and an independent set, refer to Figure 1. A *clique* is a maximal connected complete subgraph of G and an *independent set* is a subset of V(G) such that no two vertices in the subset are adjacent. The vertices in the independent set are called cones. In this paper, we are working with split graphs whose cone vertices all have the same degree, or a *proper* split graphs. Further, a split graph is *ideal* if every node in the clique is adjacent to the same number of cones. We define an y-Ideal Proper Split Graph, $y\text{−}IPS(yc_o, y, b)$, as split graphs with yc_o cones, each of degree y, having an y-grouping of cone nodes adjacent to the same y clique nodes, and b clique nodes not adjacent to any cones, refer to Figure 1. In this paper, we are concerned with $y\text{−}IPS(yc_o, y, 0)^{\mu}$ multigraphs, where all the edges within the clique are of multiplicity μ.

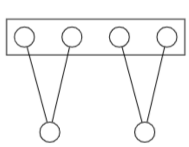

Figure 1: Ideal Proper Split Graph $2\text{-}IPS(4,2,0)^0$

One of our considerations is to analyze the graph models that maximize the number of triangles (for triangulation) between satellite and ground station with minimal cost.

CONJECTURE

A $y - IPS(yc_o, y)^{\mu}$ multigraph has the following formula for finding the number of triangles: $(y\text{-}1)/2 * y^2 * c_o * \mu_o$. Further, a $y - IPS(yc_o, y)^{\mu}$ has the following eigenvalue spectrum:

1. $c_o(y\text{-}1)$ eigenvalues equaling 0
2. $c_o(y\text{-}1)$ eigenvalues equaling $\text{-}\mu$
3. 2 eigenvalues that are the roots of $\alpha^2 + \mu\alpha - (y^2)$
4. Eigenvalues, with multiplicity of 1, that are the roots of $\alpha^2 - \mu(c_o - 1)y - (y^2)$

EXAMPLE

The goal of our research is to create a formula for the number of triangles between ground stations and satellites. This information could then be applied to satellite communications.

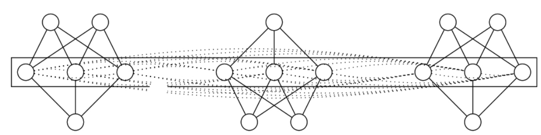

Figure 2: $3\text{-}IPS(9, 3, 0)^1$

The nodes in the box (known as a clique) represent the ground stations, while the nodes outside of the box represent the satellites. All of the ground stations connect to satellites and each ground stations connects to another once. However, the satellites only transmit signals to the ground stations and not to any other satellites.

$$\begin{pmatrix}
0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1\\
1&1&1&0&0&0&0&0&0&0&1&1&1&1&1&1&1&1\\
1&1&1&0&0&0&0&0&0&1&0&1&1&1&1&1&1&1\\
1&1&1&0&0&0&0&0&0&1&1&0&1&1&1&1&1&1\\
0&0&0&1&1&1&0&0&0&1&1&1&0&1&1&1&1&1\\
0&0&0&1&1&1&0&0&0&1&1&1&1&0&1&1&1&1\\
0&0&0&1&1&1&0&0&0&1&1&1&1&1&0&1&1&1\\
0&0&0&0&0&0&1&1&1&1&1&1&1&1&1&0&1&1\\
0&0&0&0&0&0&1&1&1&1&1&1&1&1&1&1&0&1\\
0&0&0&0&0&0&1&1&1&1&1&1&1&1&1&1&1&0
\end{pmatrix}$$

Figure 3: Adjacency Matrix for $3\text{-}IPS(9,3,0)^1$

The multigraph from Figure 2 contains 111 triangles. However, we are not concerned with the triangles of the ground stations connected to other ground stations, so we subtract that number, 84, from the total. We are left with 27 triangles from satellites connecting to ground stations.