Optimal Control Design and Implementation framework for Critical Infrastructure

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Abstract

This work considers development of control algorithms that deal with the failure of important or critical infrastructures. Critical infrastructure truly is the foundation of our modern society. A few examples of critical infrastructures are the electrical or national grid, utilities network, transportation network or even the communication network. While approaching this project, the idea of the importance of the national power grid seemed to be the main focus of development for control algorithms capable of resilience in the face of malicious attacks or disruption of efficiency from something like natural disasters, human errors and environmental changes; resource utilization; and enabling robust operation of this infrastructure system. Many different previously used algorithms such as the Gauss-Seidel algorithm showed good promise for these particular systems. With this in mind, the approach is to model the national grid infrastructure as a similar control system with linear matrix inequality system problem structure.

Summary

When designing solutions to problems in control theory, we must consider multivariable systems and state them in terms of linear matrix inequalities (LMIs) which can be closely intertwined with controller design problems [1].

We can further leverage results from modern studies considering optimization algorithms with the use of Integral Quadratic Constraints (IQCs) to provide sufficient conditions based upon the previous LMI structure discussed [2].

In real-time control situations, control systems can be represented as the feedback interconnection of both a linear time invariant system as well as a block of decentralized nonlinearities [3].

Considering the optimality of the steady-state control, the approach is mostly for linear time-invariant systems. Proposed design indicates Karush-Kuhn-Tucker optimality conditions in the steady-state [4]. However, this is without the incorporation of dual variables into said controller.

Theoretical Design and Plans

Lemma 2.3.1 ($\mathscr{L}_2$ Gain for LTI Systems) *For the LTI system*

$$G(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right],$$

with A Hurwitz, the following are equivalent:

(i) The $\mathscr{L}_2$ gain of G is less than γ;

(ii) There exists a positive definite symmetric matrix P such that

$$\begin{bmatrix} A^\mathsf{T}P + PA & PB \\ B^\mathsf{T}P & -\gamma I \end{bmatrix} + \frac{1}{\gamma}\begin{bmatrix} C & D \end{bmatrix}^\mathsf{T}\begin{bmatrix} C & D \end{bmatrix} \prec \mathbb{0}.$$

Theorem 2.1.2 (Karush-Kuhn-Tucker) *Assume a Slater point exists for the convex optimization problem (2.5). The point $x^\star \in \mathbb{R}^n$ is a global minimizer for the problem (2.5)*

if and only if there exist $\mu^\star \in \mathbb{R}^{n_{ec}}$ and $\nu^\star \in \mathbb{R}^{n_{ic}}$ such that

$$\begin{aligned} \mathbb{0} &= \nabla f(x^\star) + A^\mathsf{T}\mu^\star + \sum_{k=1}^{m} \nu_i^\star \nabla g_i(x) \\ \nu_i^\star &\geq 0, \quad i \in \{1,\dots,n_{ec}\} \\ \nu_i^\star g_i(x^\star) &= 0, \quad i \in \{1,\dots,n_{ic}\}. \end{aligned} \tag{2.8}$$

△

We consider an LTI system

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + B_w w \\ y_k &= Cx_k + Du_k + Qw \end{aligned} \tag{A.1}$$

where $A \in \mathbb{R}^{n\times n}$ is Schur, $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^p$ is the optimization output. The vector $w \in \mathbb{R}^{n_w}$ is a constant disturbance. The transfer function from $u \to y$ is

$$P(z) \coloneqq C(zI - A)^{-1}B + D$$

and from $w \to y$ is

$$P_w(z) \coloneqq C(zI - A)^{-1}B_w + Q.$$

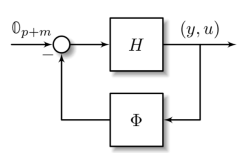
In steady-state, we have the relations

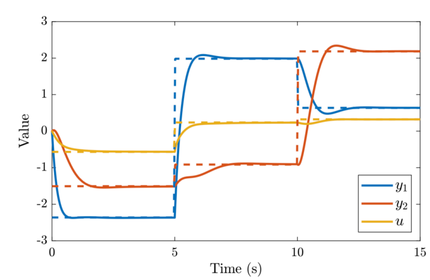
$$\bar{y} = P(1)\bar{u} + P_w(1)w.$$

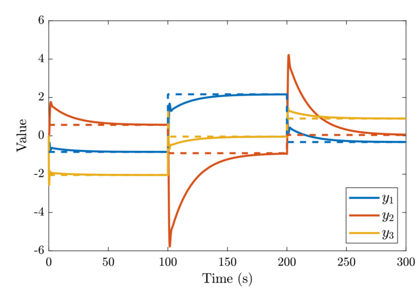
[1]The contents of this appendix section may be incorporated into a future publication: J. W. Simpson-Porco and L. S. P. Lawrence, "Stability of Discrete-Time Optimal Steady-State Controllers," publication venue to be determined.

Pictured above are the LTI conditions defined in many LMIs as well as the KKT conditions for any given control system along with an actual defined LTI system. Below, we see the pictured LTI system as well the graph of the output variables for this example , both optimized and not optimized.

$$H = \left[\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{C} & \mathcal{D} \end{array}\right] \coloneqq \left[\begin{array}{cc|c} A & BK_I & BK_P R^\mathsf{T} \\ \mathbb{0} & \mathbb{0} & R^\mathsf{T} \\ \hline C & \mathbb{0} & \mathbb{0} \\ \mathbb{0} & K_I & K_P R^\mathsf{T} \end{array}\right].$$







Results

Due to the extenuating circumstances of the coronavirus pandemic, results are sparse in this type of situation because of the lack of availability and time in the lab to work with the programmable controller. So, the results are essentially theoretical and the report focuses on the plans and models for the future plans and models, which is what most of the time was spent on anyway.

Future Directions

This work focuses primarily on use of a modified Projected Gauss-Seidel (PGS) algorithm with various input in the form of a Linear Matrix Inequality which can deal with both nonlinear as well as linear time invariant systems but will more so focus nonlinear systems. Considering the presented work and theoretical plans, future work and plans are essential in the continuation of this piece. The report introduces ways to form a unique framework of a nonlinear controller that drives a given an optimal solution given various different and unique inputs. The main focus in the future will be to add conditions sufficient on said controller as well as adding constraints that will best represent and solve problems modeling the complexity of the national grid. The most important thing to consider is that the LMI conditions are still satisfied with the addition of these conditions and constraints and that the algorithm is robust and efficient enough to apply to actual critical infrastructure essential to daily life in society.

References

[1] L. S. P. Lawrence, "The Optimal Steady-State Control Problem," *Thesis presented to the University of Waterloo in fulfillment of the thesis requirement for the degree*, 2019.

[2] Z. E. Nelson and E. Mallada, "An integral quadratic constraint framework for real-time steady-state optimization of linear time-invariant systems," *2018 Annual American Control Conference (ACC)*, June 27-29, 2018.

[3] A. A. Adegbege and Z. E. Nelson, "A Gauss-Seidel type solver for the fast computation of input-constrained control systems," *Elsevier B.V.*, 2016.

[4] L. S. P. Lawrence, Z. E. Nelson, E. Mallada and J. W. Simpson-Porco, "Optimal Steady-State Control for Linear Time Invariant Systems," *IEEE Conference on Decision and Control (CDC)*, Dec. 17-19, 2018.