Investigation of Chaotic Behavior of a Non-linear Pendulum

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Introduction

In order to study chaos theory a mechanical system was made to demonstrate the phenomenon. The mechanical system is driven by an oscillator and includes varying lengths of springs and masses. Photogates and rotary motion sensors are used for the collection of data from the mechanical system. Phase plots and Poincaré diagrams were created using the program Mathematica.

Theory

Chaos theory was first observed by meteorologist Edward Lorenz in 1961 while creating a predictive model of a weather event. He changed the precision of the model to include another decimal place; this caused the model to change drastically and led him to the formulation of the phenomenon. The phenomenon was first called chaos theory in 1963; it is defined as the dramatic difference in long term end states resulting from a slight change in the initial conditions of a system.

One of the best ways to evaluate whether a system is chaotic or not is to use a phase plot. A phase plot is a representation of the trajectories of a dynamic system in motion. The displacement is plotted against its derivative with respect to time (velocity). If the data is spread out over the entire graph, chaotic motion is present.

The plot used in our experiment graphs the angle (in radians) vs the angular velocity (in radians/second) to produce the phase plot.

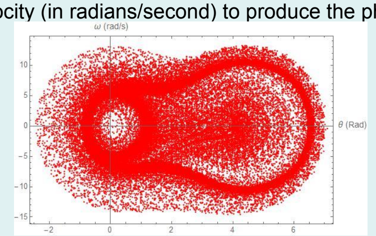

Figure 1: Phase plot of experimental data (77,530 data points taken over a 60 minute time period) with driving frequency 3.56 Hz, and driving amplitude 3.86 cm. Chaotic behavior is seen on the plot as a spreading out of data over the entire range of the plot.

Another useful tool to observe chaos is the Poincaré plot. A Poincaré plot is a two-dimensional graph that displays the time-based dynamic behavior of a system. Poincaré plots are subsets of phase plots and use data points that are sampled at a set time interval. Using a Poincaré plot, order within chaos can be seen. Poincaré plots are used often in chaotic systems to find attractors (places in which the system will be found more often in).

Experimental Apparatus

The apparatus is a pendulum that uses a motor and oscillating arm to generate motion in the system. The pendulum is connected to two springs on either side of it (Figure 2); the length of the springs can be adjusted to help generate chaotic motion.

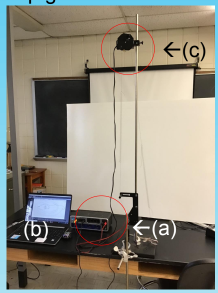

Figure 2

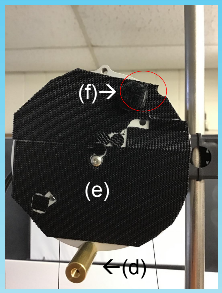

Figure 3

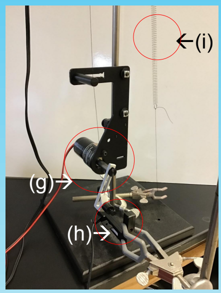

Figure 4

Figure 2 shows the PASCO interface (a) and laptop computer (b) needed to operate in using the PASCO program; the pendulum and rotary motion sensor is (c). Figure 3 shows the offset weight (d) of the pendulum attached to a disk plate covered in Velcro (e) which is free to rotate at its center. The disc plate is covered in Velcro to make the placement of Velcro-covered magnets (f) easier. Another magnet is attached to the rotary motion sensor in the back of the disc (not visible in figure). In Figure 4 the mechanical oscillator (g), photogate (h) and one of the spring (i) are shown.

The movable Velcro-covered magnet was used in the apparatus to provide a perturbing force which would cause the motion of the pendulum to develop chaotic or nonchaotic motion. The interaction between the movable Velcro-covered magnet and the fixed magnet (on the rotary motion sensor) causes this perturbing force. The initial conditions of the apparatus was also adjusted by changing the length of the springs and the amplitude and frequency of oscillation. Chaotic behavior was observed when the magnet was placed on the disk at the furthest point from the weight. This allowed the pendulum to move faster and rotate more than 360 degrees.

Experimental Results

Using the PASCO Capstone computer program data was collected; the displacement angle was plotted versus the corresponding angular velocity. This phase plot is shown in Figure 5. In chaotic motion the phase plot is filled in and motion does not repeat itself.

Figure 6 is the Poincaré plot corresponding to the phase plot shown in Figure 5. Notice that there is an underlying pattern in the Poincaré plot even when there is chaotic motion present.

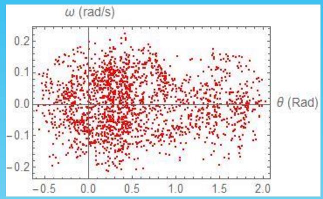

Figure 5

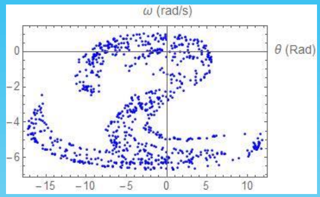

Figure 6

Non-chaotic motion in our system looks very different from chaotic motion on a phase plot. For non-chaotic motion the pendulum repeats itself in a predictable pattern (Figure 7). For most of the data sets the system displayed periodic motion; the amplitude and frequency of oscillation.

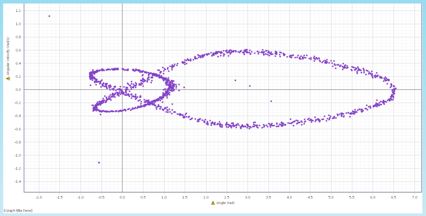

Figure 7: Phase plot showing the pendulum undergoing periodic motion.

Conclusion

In this project we were able to generate chaotic motion in a pendulum system. We were able to demonstrate that small perturbations were able to cause the system to go from periodic motion to chaotic motion. We were able to construct Poincaré plots that demonstrated attractors in our system. Future work that can be explored on this topic is the modelling of differential equations, approximating the solutions using numerical methods, plotting the answers, and comparing them to the experimental plots.

References

University of Florida. (2015). Chaotic Pendulum [PDF file]. Retrieved from https://www.phys.ufl.edu/courses/phy4803L/group_IV/chaos/Chaos.pdf

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