Department of Mechanical
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Mathematical Modeling of Corneal Indentation

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Abstract

The goal of this research is to develop a reduced order mathematical mechanical model to accurately capture the non-linear load-deflection behavior of a cornea undergoing indentation. Toward this end, equations were formulated and results based on the mathematical model were compared to published experimental data. The model was seen to accurately predict quantitative as well as qualitative behavior of an indented cornea.

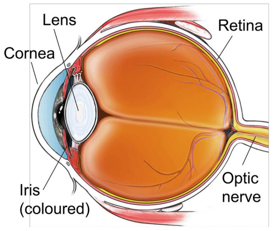

Image from www.healthdirect.gov.au/surgery/corneal-transplant-surgery.

Mathematical Models

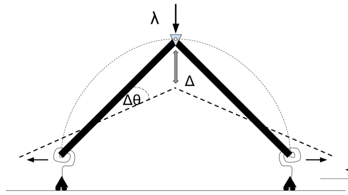

Figure 1: Torsional Spring Model

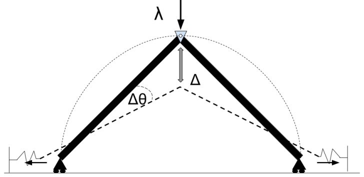

Figure 2: Horizontal Spring Model

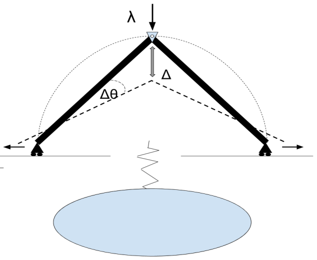

Figure 3: Vertical Linear Spring Model

Two models were initially considered: the Torsional Spring and Rod model (Figure 1) and the Horizontal Spring and Rod Model (Figure 2). Governing equation s were derived for each model from work-energy principles. In each case, the indenting force, λ, is the independent variable and the crown-point deflection, Δ, is the dependent variable, yielding a load-deflection path of equilibrium configurations. A third model (Figure 3) was additionally proposed to represent the effect of the lens on the indentation, and could be combined with either Model 1 or 2. Results of these models were graphed and compared to the published results of experimental indentation tests of Delori et al (1969) (Figure 4). The stiffness values were found by estimating the energy (area under the load-deflection curve) and substituting that value into the pertinent governing equation. Models were combined and values of the linear and nonlinear stiffness were assessed to match the experimental data.

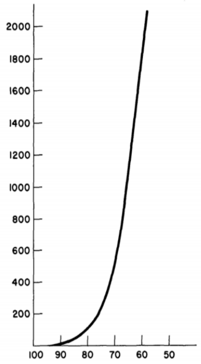

Figure 4: Load-deflection data of the eye by Delori et al (1969).

Background

The cornea is a transparent tissue covering the pupil of the eye. The main functions of the cornea include refracting incoming light rays into the lens and protecting tissues inside of the eye from outside particles. The purpose of a reduced order model is to effectively approximate the behavior of the cornea during indentation without the use of complex representation such as shell theory. The dimensions of the cornea were found in the literature to assess corresponding dimensions of the proposed models. The typical rise of the cornea is 3.77 mm and the typical radius of the cornea where it connects with the sclera is 7.5 mm (Ritzmann et al, 2018). This indicates that the effective "rods" of the proposed mechanical models have a length of 8.39 mm. It is pertinent to point out that while the models may appear two dimensional, the model consists of an assembly of rigid rods situated about the anterior-posterior axis of the eye, forming a cone-like structure. In addition, the sclera is considered rigid in comparison to the cornea.

Acknowledgements

I would like to thank the New Jersey Space Grant Consortium for giving me this wonderful opportunity to have an educational summer and I look forward to continuing my research.

Analysis and Results

The model that best fit the load-deflection data of the entire eye presented by Delori et al (1969) consisted of a combination of nonlinear torsional spring and nonlinear vertical spring (Figures 1 and 3) (graph shown in Figure 5). However, it was necessary to confirm whether the lens contributed to the resistance of the deflection as well. Results of lens indentation tests of Ulrich et al (2016) (Figure 5) showed that the vertical spring model did not adequately represent the influence of the lens. Comparison with results of the corneal indentation test of Ahearne et al (2007) showed that a combination of linear torsional and nonlinear torsional springs (Figure 1) modeled the behavior of the cornea alone very well. As seen in Figure 6, when compared with the results of Delori et al (1969), the model correctly predicts the load-defection behavior until the crown-point deflection reaches 4-5 mm. This suggests that eye tissue in addition to the lens and the cornea starts to resist the indenter at that deflection level.

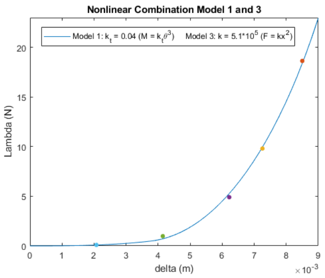

Figure 5: Load-deflection curve of model (solid line) fitting selected data points (dots) from Delori et al (1969)

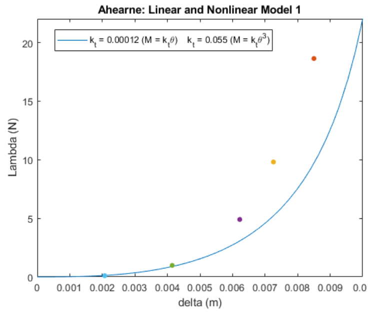

Figure 6: Load-deflection curve of model fitting data of Ahearne et al (2007) (solid line) , compared with selected data points (dots) from Delori et al (1969)

Future Direction

In the future, this model will be expanded to include inertial effects and the response to impact. The intent is that this mathematical model will ultimately be coupled with a rigorous visco-elastodynamic model of the complete eye. This dynamic model will eventually be used to study the influence of trauma on eye damage – specifically retinal detachment.

References

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