LYAPUNOV-BASED ATTITUDE CONTROL OF FREE-FLOATING SATELLITES WITH ONBOARD ROBOTIC MANIPULATORS

Ethan M. Catalanello and Annalisa Scacchioli (Faculty Advisor)

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Introduction and Project Goals

In controls engineering, Lyapunov Control Theory is regarded as a highly effective method of designing controllers as a result of its strict stability definition as well as its ability to create very robust controllers that can exhibit a high degree of performance despite minor inaccuracies being present in the model of the system. This makes Lyapunov controllers a very attractive option for those designing spacecraft that will be subjected to unmodelled external disturbances while necessitating very little error. Thus, this project explores the effectiveness of a Lyapunov controller on a spacecraft with onboard robotic manipulators. The goal is to design a nonlinear Lyapunov controller that can stabilize the satellite throughout the operation of its manipulators.

Background

As humanity extends its reach further into space, one of the biggest challenges it will face is the development of Earth-based space infrastructure. Currently, much of the modern lifestyle, including GPS navigation, telecommunication, and internet relies on orbital satellites. As humanity continues to modernize and expand in accordance with NASA's Space Technology mission directorate, it is highly probable that its dependence on these and similar systems will increase.

This is problematic because unlike systems on Earth, no method of diagnosis or repair exists for space-based infrastructure. Normally, when an anomaly or dysfunction is experienced on the ground, technicians can not only diagnose the problem, but also repair the system with minimal downtime. This is not the case in space. When satellites in orbit experience an anomaly, even if the source of the problem is determined, it is nearly impossible to initiate any repairs.

In response to this The Defense Advanced Research Projects Agency (DARPA) has created a proposal for a robotic, autonomous servicing spacecraft that is designed to reside in geostationary orbit [1]. When a geosynchronous satellite experiences an anomaly, the DARPA satellite will be dispatched to the ailed satellite, and use its various onboard tools, such as dual robotic manipulators, to diagnose and repair the geosynchronous satellite.

Such a design serves as a complex controls engineering challenge that requires extensive attention to ensure the satellite remains fixed despite the motions of its manipulators.

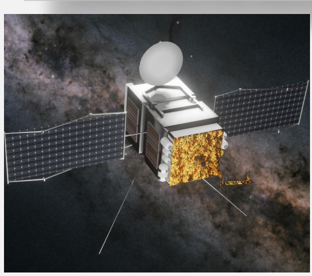


Figure 1 - Artist's conception of DARPA's RSGS Satellite design.

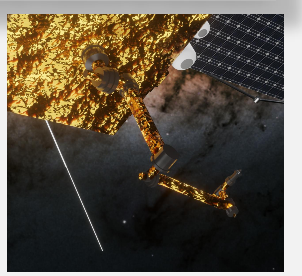


Figure 2 - Artist's conception of DARPA's FREND Mk. II mounted on a satellite.

Methodology

Dynamics Modelling

For this simulation, the satellite was assumed to be a rigid body, and thus had its motion governed by Euler's Equation of Rotational Dynamics (shown in Equation 1). Note that we are assuming a body-fixed coordinate frame.

$$[\boldsymbol{I}]\dot{\boldsymbol{\omega}} = -[\tilde{\boldsymbol{\omega}}][\boldsymbol{I}]\boldsymbol{\omega} + \boldsymbol{L}_c + \boldsymbol{Q} \quad (1)$$

Equation 1 – Euler's Equation of Rotational Motion for rigid bodies.

The system's attitude is modelled using both Directional Cosine Matrices (DCMs) and Modified Rodrigues Parameters (MRPs), as they both have attractive properties for different applications. The dynamics integration is done using DCMs, as they have a normalization property that allows them to retain accuracy throughout numerical integration. The differential kinematics equation for DCMs is shown in Equation 2.

$$[\dot{BN}] = -[\tilde{\boldsymbol{\omega}}][BN] \quad (2)$$

Equation 2 – DCM Differential Kinematics Equation

Lyapunov Controller Development

In order to apply Lyapunov Theory, a positive-definite Lyapunov potential function was created for the system (shown in Equation 3 [2]). Note that attitude is represented in terms of MRPs here, as they not only linearize very well (which is helpful for gain selection) but also lead to a very simple feedback control law.

$$V(\boldsymbol{\sigma}, \delta\boldsymbol{\omega}) = \frac{1}{2}\delta\boldsymbol{\omega}^T[I]\delta\boldsymbol{\omega} + 2K\ln(1 + \boldsymbol{\sigma}^T\boldsymbol{\sigma}) \quad (3)$$

Equation 3 – Lyapunov function

After completing the necessary proofs for asymptotic and global stability, a nonlinear control law with two gains (shown in Equation 4) can be derived for the satellite system. Gain selection was completed through plant linearization.

$$\boldsymbol{Q} = [\tilde{\boldsymbol{\omega}}][\boldsymbol{I}]\boldsymbol{\omega} - \boldsymbol{L}_c - [P]\boldsymbol{\omega} - K\boldsymbol{\sigma} \quad (4)$$

Equation 4 – Feedback control law for satellite

Variable	Description
$[BN]$	DCM transforming coordinates from frame $\mathcal{N}$ into frame $\mathcal{B}$.
$\boldsymbol{\sigma}$	Attitude (MRP)
$\boldsymbol{\omega}$	Satellite angular velocity (body frame)
$\delta\boldsymbol{\omega}$	Difference in angular velocity from reference (body frame)
$[\boldsymbol{I}]$	Satellite inertia tensor
K	Selectable positive scalar gain
$[P]$	Selectable positive definite matrix
$\boldsymbol{L}_c$	External torque (can be disturbance)
$\boldsymbol{Q}$	Control torque

$$\boldsymbol{\sigma} = \begin{pmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\end{pmatrix} = \frac{1}{\zeta(\zeta+2)}\begin{pmatrix}C_{23} - C_{32}\\ C_{31} - C_{13}\\ C_{12} - C_{21}\end{pmatrix} \quad \zeta = \sqrt{trace([C]) + 1} \; (5)$$

$$[C] = [\boldsymbol{I}_{3\times3}] + \frac{8[\tilde{\boldsymbol{\sigma}}]^2 - 4(1 - |\boldsymbol{\sigma}|^2)[\tilde{\boldsymbol{\sigma}}]}{(1 + |\boldsymbol{\sigma}|^2)^2} \; (6)$$

Equations 5 and 6 – Conversions between MRP ($\boldsymbol{\sigma}$) and DCM ($[C]$)

Results and Discussion

The satellite was subjected to the disturbance of a simulated robotic arm load that was developed during the Spring 2020 semester. This load was based on a numerical model of DARPA's FREND Mk. II arm, which was designed to be used on its RSGS Spacecraft. Figure 3 shows the block diagram of the system, while Figure 4 compares the controlled between the newly-developed Lyapunov Controller and the previously utilized PID controller. The disturbance was estimated by multiplying each moment load by a Gaussian randomly distributed scaling factor to simulate measurement error, estimation inaccuracy, and sensor noise.

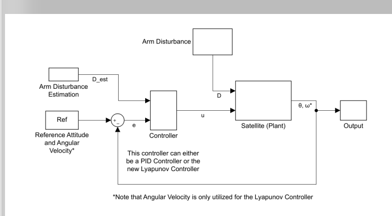


Figure 3 (Above) – Block diagram of the satellite-controller system. Note that both controllers require an estimation of the arm's disturbance as an input; however, only the Lyapunov controller requires the system's current angular velocity.

Figure 4 (Center) – Comparison of closed loop responses between the nonlinear Lyapunov controller and linear PID controller when the system was subjected to the simulated robotic arm load. Note how the Lyapunov system has a much lower peak error and a much faster response time when compared to the PID controller. From this, we conclude that the Lyapunov controller is much more robust and effective at controlling the satellite than the linear PID system.

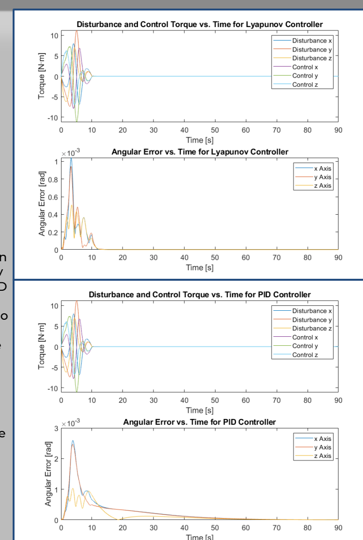


Figure 5 (Right) – The new Lyapunov controller was also compared with the PID controller in a worst-case scenario setting where the satellite is in a tumble. The initial conditions are as follows:

$$\boldsymbol{\theta}_0 = \left[\frac{\pi}{3}, -\frac{\pi}{4}, \frac{\pi}{6}\right] rad$$

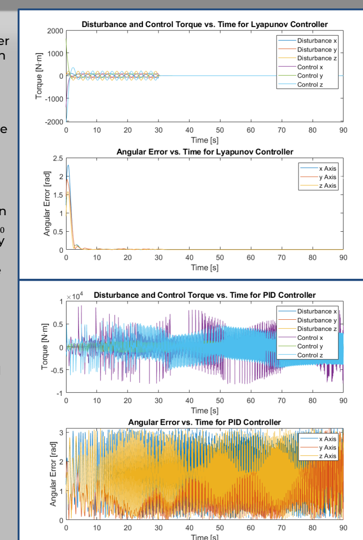
$$\boldsymbol{\omega}_0 = [1, -3, -5]\frac{rad}{s}$$

Where θ_0 is measured in ZYX Euler angles and $\boldsymbol{\omega}_0$ is measured in the body frame.

The disturbance torque was as follows:

$$L_1 = 100\sin(t)$$
$$L_3 = 100\cos(t)$$
$$L_2 = \frac{L_1 L_3}{25}$$
$$L(t) = [L_1, L_2, L_3]$$

The torque was applied for the first 30 seconds. Notice how quickly the Lyapunov controller recovered from the tumble and how effective it remained throughout the disturbance, while the PID controller failed to stabilize the system at all.



Future Directions

- Incorporate models of more realistic actuators and sensors to more accurately evaluate the performance of the system
- Redesign the controller to feature more adaptive behavior

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