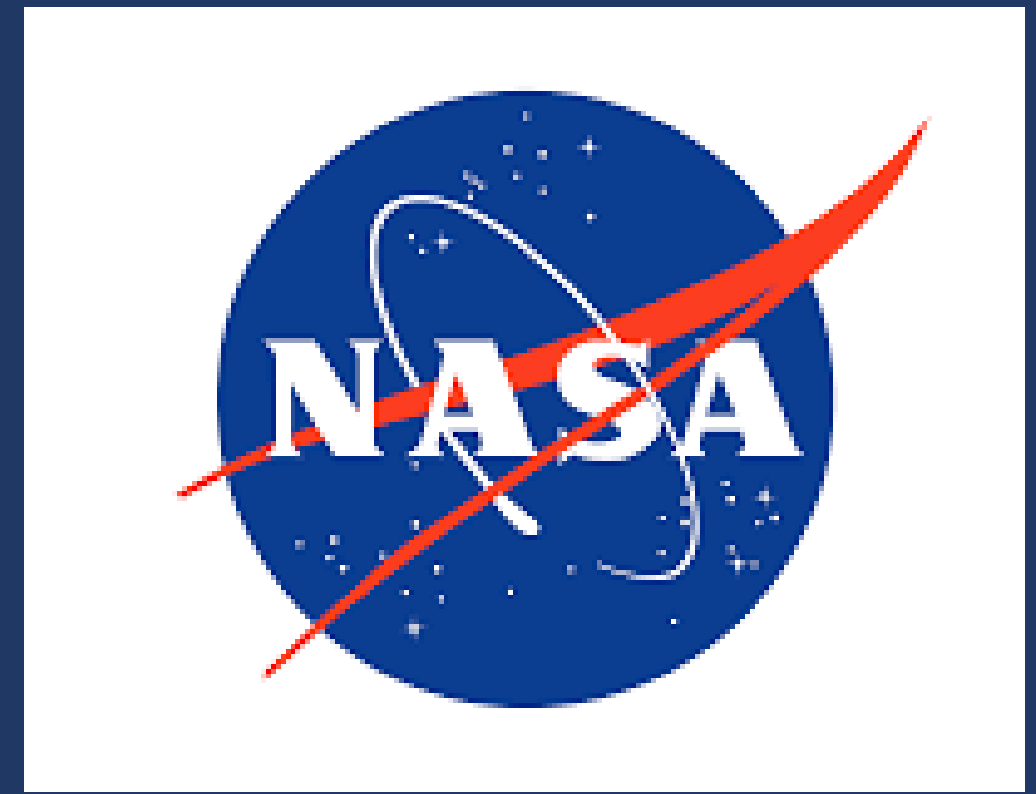


# Analog Circuits for Embedded Model Predictive Control

Francis Moran, Dr. Ambrose Adegbege  
 The College of New Jersey School of Engineering  
 New Jersey Space Grant Consortium



## Introduction

Embedded analog circuits have been looked in to due to their ability to efficiently and accurately perform computations relating to control. However, there has yet to be a comprehensive study of control implementations of these circuits. Some of these possible controllers include proportional integral, low gain integral low gain quadratic programming, and model predictive controllers (MPC). This past summer I have worked on implementing these controllers and testing them on the quadruple tank control testbed.

## Model Predictive Control

Although many other controllers were implemented, the focus of this research was on Model Predictive Control. This method of control aims to minimize a quadratic programming problem to find an ideal control output. MPC also incorporates constraints into the minimization. These constraints come from real life data and help push the plant to its limits, resulting in optimized control. Below is the minimization problem used.

$$\min_u \frac{1}{2} U^T H U + U^T q$$

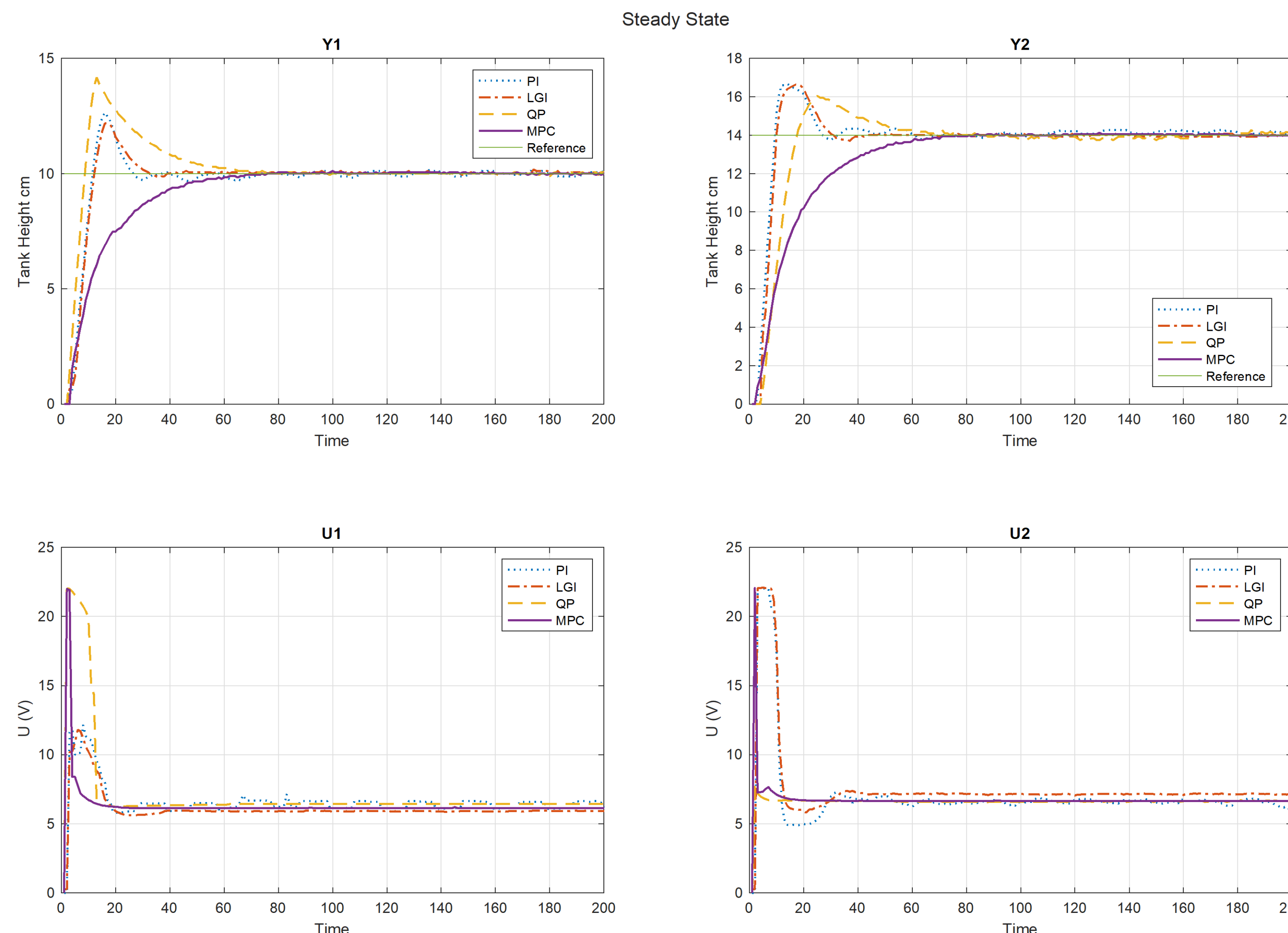
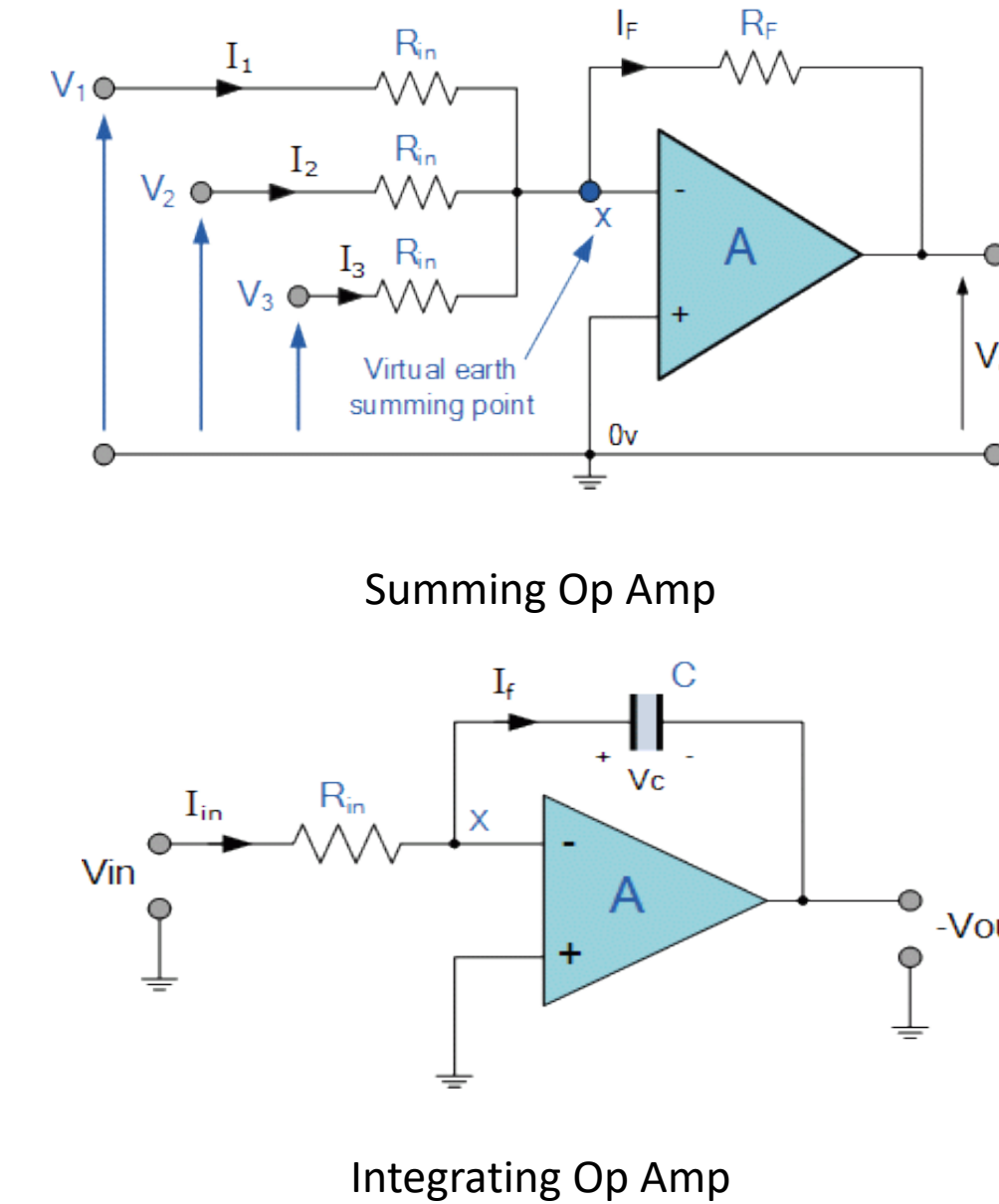
## Implementation of Minimization Problem

While the minimization problem shows the goal of the controller, it must be converted into a differential equation to be implemented as an analog circuit. Using gradient dynamics, the following differential equation is realized:

$$\dot{U} = -U + sat(U - HU - q)$$

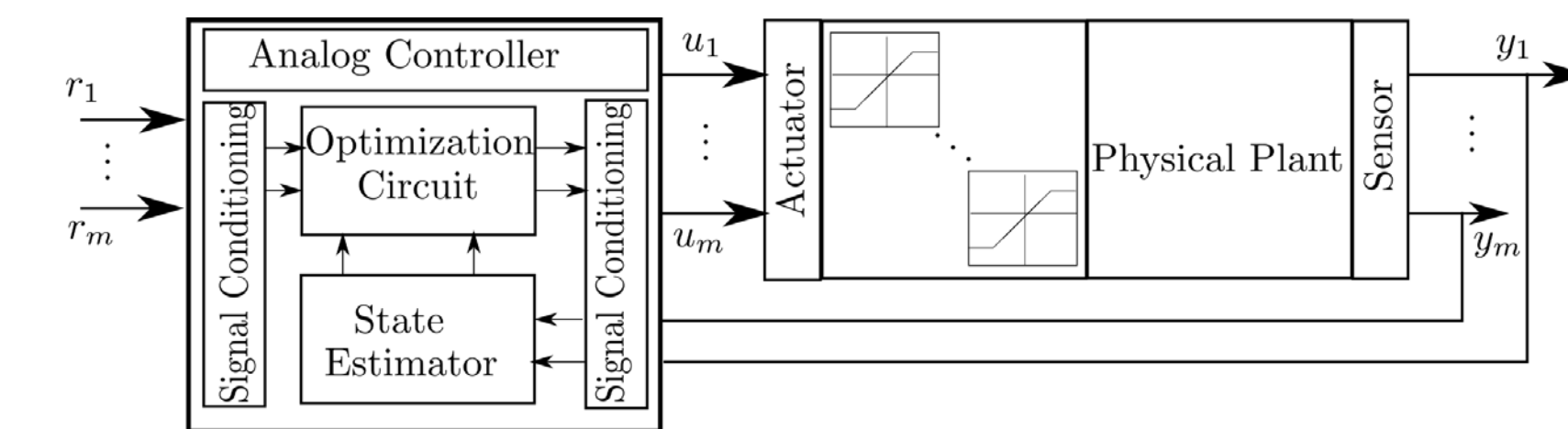
## Analog Implementation

In order to effectively implement this differential equation as an analog circuit, it is necessary to break it down to its arithmetic components. This includes summing, integration, and saturation. All these operations can be implemented using a combination of op amps, resistors, capacitors, and diodes, all of which are common analog devices. Examples of these circuits are shown to the right. However, in order to streamline all our circuits, a field programmable analog array (FPAA) was used. An FPAA allows an analog circuit to be programmed and downloaded onto a chip for fast and accurate prototyping.



## Experimental Setup and Results

As mentioned previously, all the testing done on these circuits was completed using the quadruple tank system. This system is comprised of the coupling of two sets of vertically positioned tanks. The control objective was to set the water levels in the bottom tanks to a desired level (seen in results as reference). The control architecture is shown below, with feedback from sensors into the controller and the controller determining the flow rate being defining features. In this experiment, all four controllers were stepped to the reference point at t=0 and run for 200 seconds. As seen in the results section, all four controllers converged to the solution, showing the working of complex analog control. The variable U represents the control output, and Y the tank height.



## Future Work

Further work would include implementing the model on a physical device. One possible candidate for the device could be the Quasar 2 DOF Helicopter. This system models air or underwater vehicles and would provide a reliable model to test all 4 controllers.

## Contacts

Francis Moran  
 Email - [moranf2@tcnj.edu](mailto:moranf2@tcnj.edu)  
 Cell - (732) 672 - 3049

Dr. Ambrose Adegbege  
 Email - [adegbega@tcnj.edu](mailto:adegbega@tcnj.edu)